Present Value of an Annuity

Finite Math

27 September 2018

Quiz

What is an annuity?

Present Value - Set Up

We will look at making a large deposit in order to have a fund which we can make constant withdraws from. We make an initial deposit, then make withdraws at the end of each interest period. We should have a balance of $0 at the end of the predetermined amount of time the fund should last.

Example

Example

How much should you deposit into an account paying 6% *compounded semiannually in order to be able to withdraw* \$2000 *every* 6 *months for* 2 *years? (At the end of the* 2 *years, there should be a balance of* \$0 *in the account.)*

Solution

This problem is solved similarly to how the future value of an annuity was, except this time, instead of finding the future value of each deposit, we have to find the present value of each withdraw. We do this because we want to only deposit enough money to be able to withdraw the $2000 at the specified time. We can collect these again in a table:

Withdraw	Term Withdrawn	Number of times Compounded	Present Value
\$2000	1	1	$\$2000\left(1+\frac{0.06}{2}\right)^{-1} = \$2000(1.03)^{-1}$
\$2000	2	2	$\$2000\left(1+\frac{0.06}{2}\right)^{-2} = \$2000(1.03)^{-2}$
\$2000	3	3	$\$2000\left(1+\frac{0.06}{2}\right)^{-3} = \$2000(1.03)^{-3}$
\$2000	4	4	$\$2000\left(1+\frac{0.06}{2}\right)^{-4} = \$2000(1.03)^{-4}$

So adding up the present values of all these will give us the amount of money we should deposit into the account now

$$D = \$2000(1.03)^{-1} + \$2000(1.03)^{-2} + \$2000(1.03)^{-3} + \$2000(1.03)^{-4} = \$7434.20$$

Present Value of an Annuity

Definition (Present Value of an Ordinary Annuity)

$$PV = PMT\frac{1-\left(1+\frac{r}{m}\right)^{-n}}{r/m}$$

where

$$\begin{aligned} PV &= \textit{present value} \\ PMT &= \textit{periodic payment} \\ r &= \textit{annual nominal interest rate} \\ m &= \textit{frequency of payments} \\ n &= \textit{number of payments (periods)} \end{aligned}$$

Note that the payments are made at the end of each period.

Now You Try It!

Example

How much should you deposit in an account paying 8% *compounded quarterly in order to receive quarterly payments of* $\$1,000$ *for the next* 4 *years?*

Solution

$\$13,577.71$

Combination Example

An interesting application of this in conjunction with sinking funds is saving for retirement.

Example

The full retirement age in the US is 67 *for people born in* 1960 *or later. Suppose you start saving for retirement at* 27 *years old and you would like to save enough to withdraw* $40,000 *per year for the next* 20 *years. If you find a retirement savings account (for example, a Roth IRA) which pays* 4% *interest compounded annually, how much will you have to deposit per year from age* 27 *until you retire in order to be able to make your desired withdraws?*

Now You Try It!

Example

Lincoln Benefit Life offered an ordinary annuity earning 6.5% *compounded annually. If* $\$2,000$ *is deposited annually for the first* 25 *years, how much can be withdrawn annually for the next* 20 *years?*

Solution

$\$10,688.87$

Amortization

Amortization is the process of paying off a debt. The formula for present value of an annuity will allow us to model the process of paying off a loan or other debt. The reason the formula is the same is because receiving payments from your savings account is essentially the bank repaying you the money you loaned them by depositing it into a savings account.

Amortization

Example

Suppose you take out a 5*-year,* \$25,000 *loan from your bank to purchase a new car. If your bank gives you* 1.9% *interest compounded monthly on the loan and you make equal monthly payments, how much will your monthly payment be?*

Now You Try It!

Example

If you sell your car to someone for $\$2,400$ *and agree to finance it at* 1% *per month on the unpaid balance, how much should you receive each month to amortize the loan in* 24 *months? How much interest will you receive?*

Solution

$PMT = \$112.98$, $I = \$311.52$

Amortization Schedule

Example

Construct the amortization schedule for a $\$1,000$ *debt that is to be amortized in six equal monthly payments at* 1.25% *interest per month on the unpaid balance.*

Amortization Schedule

Payment Number	Payment	Interest	Unpaid Balance Reduction	Unpaid Balance
0				$\$1,000$
1	$\$174.03$	$\$12.50$	$\$161.53$	$\$838.47$
2	$\$174.03$	$\$10.48$	$\$163.55$	$\$674.92$
3	$\$174.03$	$\$8.44$	$\$165.59$	$\$509.33$
4	$\$174.03$	$\$6.37$	$\$167.66$	$\$341.67$
5	$\$174.03$	$\$4.27$	$\$169.76$	$\$171.91$
6	$\$174.06$	$\$2.15$	$\$171.91$	$\$0.00$
Total	$\$1,044.21$	$\$44.21$	$\$1,000$	